As the above discussion is limited to uniaxial tensile tests, it would be premature to reach any general conclusions about carbide cracking in pearlite as an emission source in all deformation situations. For uniaxial tensile stress conditions, Miller and Smith [12] have proposed a crack propagation mechanism whereby the cracking of individual carbide plates extends progressively throughout a pearlite colony, via slip in the interplate ferrite. However, if such a process, from the AE standpoint, occurs slowly the unit emission source will be the fracture of individual carbide lamellae. On the other hand, the general stress field around a colony could affect such a deformation process. For example, in crack propagation tests, stress triaxially in the crack tip region may promote the pearlite to fail in such a way that many individual carbide plates fracture simultaneously. Under these conditions the unit emission process would not be the same as that described above. Thus the emission characteristics of one particular microstructural feature may well be different in different stress field configurations.

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Received 16 March and accepted 28 April 1977

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Prediction of thermal fatigue life of ceramics

Crack growth in one cycle of thermal stress can not be given easily due to the difficulty in knowing the precise thermal stress distribution. This seems to make it difficult to relate the results of short-term tests to the fatigue life in practice. However, consideration of the fact that the fatigue life is approximated by the duration in which the crack length reaches a certain critical value, beyond which the growth becomes relatively rapid, may make the prediction easy.

The slow crack growth can be described by

$$\frac{da}{dt} = AK_I^n \qquad (1)$$

where a, t, K_I, A and n stand for crack length, time, stress intensity factor and material constant, respectively [1]. K_I is, also, expressed by

$$K_I = Ya^{1/2}\sigma \qquad (2)$$

where Y and σ stand for geometrical factor and applied stress, respectively [2]. Substitution of Equation 2 into Equation 1 gives,

$$\frac{da}{dt} = AY^n a^{n/2}\sigma^n \qquad (3)$$

σ, in general, consists of mechanical stress, σ_M, and thermal one, σ_T. When thermal fatigue limits the life of a ceramics, σ_M can be neglected and

$$\sigma \approx \sigma_T. \qquad (4)$$

Under thermal stress, generally, the stress distribution and the type of the stress change with the crack length as well as time, depending on the shape of the ceramic articles and heating or cooling process. Thus, in general, Y and σ depend on the crack length as well as the time and temperature, which makes it difficult to obtain the analytical solution for a in Equation 3. For the prediction, however, the following approximation can be made for Y and σ.

The growth rate of a crack is proportional to $a^{n/2}$ ($n \geqslant 20$), and σ as well as Y is not so a rapidly decreasing function of a. So, the growth rate can be expected to become rapid as the crack grows. Thus, the substantial part of the fatigue life is expected to have passed before the crack length reaches a certain critical value, a_c.

The critical length is generally thought to be small. For example, for a ceramic of $n = 20$, on the assumption that Y and σ are not so rapidly decreasing functions, the ratio of the crack velocity, v_1, at $a = 10\,\mu$m to that, v_2, at $a = 20\,\mu$m is about 10^{-3}. The time (or number of cycles of thermal shock), t_1, in which the crack reaches the depth of $10\,\mu$m from the surface is longer than $10\,\mu\text{m}/v_1$.

The time, t_2, in which the crack reaches the depth of $10^4\,\mu$m from that of $20\,\mu$m, is shorter than $10^4\,\mu\text{m}/v_2$. Therefore, $t_1 > t_2$, which shows that the critical length can be taken as about 20 μm. Variation of σ and Y would not change the value substantially.

As seen above, for prediction of the fatigue life of ceramics, the crack growth can be restricted within a small range of a. Thus, in Equation 3, Y and σ can be approximated to be constant,

$$\begin{aligned} Y &= Y(a = 0, \quad t) \\ \sigma &= \sigma\,(a = 0, \quad t) \end{aligned} \tag{5}$$

Moreover, σ_T is directly proportional to the temperature difference in the thermal shock, when heating or cooling condition is kept unchanged except for the value of the temperature difference, ΔT. Thus,

$$\sigma_T \approx k\Delta T f(t), \tag{6}$$

where k is a constant. After substitution of Equations 5 and 6 into Equation 3, integration gives

$$a_i^{(2-n)/2} - a^{(2-n)/2} = \left(\frac{n-2}{2}\right) AK^n(\Delta T)^n \times \int_0^t Y^n f^n \mathrm{d}t \tag{7}$$

where a_i stands for the initial crack length. For one cycle of the thermal shock, the upper limit t of the integration in Equation 7 is taken as infinity. Thus, Equation 7 results in

$$a_i^{(2-n)/2} - a^{(2-n)/2} = \left(\frac{n-2}{2}\right)(\Delta T)^n G \tag{8}$$

where

$$G = k^n A \int_0^\infty Y^n f^n \mathrm{d}t.$$

For M times of thermal shock,

$$a_i^{(2-n)/2} - a^{(2-n)/2} = \left(\frac{n-2}{2}\right)(\Delta T)^n MG \tag{9}$$

The thermal fatigue life N is determined by equating a to a_c, in Equation 9. For ceramics of the same a_i, the following relation is easily derived,

$$N(\Delta T_N)^n = N'(\Delta T_{N'})^n, \tag{10}$$

where ΔT_N and $\Delta T_{N'}$ correspond respectively to the severity of the thermal shock, for which the fatigue life is N and N'.

Equation 10 gives also,

$$(N/N') = (\Delta T_{N'}/\Delta T_N)^n. \tag{11}$$

Equation 10 or 11 holds for the median or mean value of the fatigue life of ceramics having the same statistical feature, and evidently relates the results of short-term tests to those of long-term tests, and therefore is useful for prediction of fatigue life.

For individual ceramics, a statistical treatment is needed. As is well known, the fracture strength of ceramics follows the Weibull equation,

$$P = \exp\left[-V(\sigma_f/\sigma_0)^m\right] \tag{12}$$

where P is the survival probability under the fracture stress σ_f, m is Weibull modulus and σ_0 is a normalizing constant, and V is the stressed volume. The fracture stress σ_f is related to a_i through the following

$$\sigma_f^n \approx l a_i^{(2-n)/2} \quad (l\text{: const.}) \tag{13}$$

where l is a constant. Thus, P is written as follows,

$$P = \exp\left[-V(a_i/\sigma_0')^{m'}\right], \tag{14}$$

where

$$\sigma_0' = \sigma_0^{2n/(2-n)}\, l^{2/(n-2)}$$

$$m' = m(2-n)/2n.$$

In Equation 9, as $n \gtrsim 20$ and $a_i < a_c$, a_i can be approximated as follows:

$$a_i^{(2-n)/2} \approx \frac{n-2}{2} N(\Delta T_N)^n G. \qquad (15)$$

Substitution of Equation 15 into Equation 14 gives,

$$P = \exp[-V' N^{m/n} (\Delta T_N)^m], \qquad (16)$$

or

$$\ln(-\ln P) = \ln V' + \frac{m}{n} \ln N + m \ln(\Delta T_N), \qquad (17)$$

where

$$V' = V l^{m/n} \left(\frac{n-2}{2}\right)^{m/n} G^{m/n} \sigma_0^{-m}.$$

Equation 16 or 17 gives the relationship between the thermal fatigue life, N, the survival probability, P, and the thermal shock severity, ΔT_N, which closely resembles the S–P–T relationships derived by Tappin, Davidge and McLaren [3].

Through Equations 11, 16 and 17, results of short-term tests can be related to those of long-term ones. The validity of Equation 11 will be discussed by using the results given by Hasselman *et al.* [4]. $\ln N - \ln(\Delta T_N)$ plots from the results [4] are given in Fig. 1. As shown in the figure, $\ln N$ is proportional to $\ln(\Delta T_N)$, which seems to prove the validity of the present theory.

The constant, n, determined from Figure 1 is about 30. Although this value of n is a little larger than that determined by other methods ($n \approx 20$), the agreement is rather good. This tendency for n, as determined by Equation 11, to be a little larger than the value determined by the crack growth measurement, is observed for a hot-pressed silicon nitride, whose N and ΔT_N are given by Ammann *et al.* [5] (In this case, the linearity between $\ln N$ and $\ln \Delta T_N$ holds). n determined by Equation 11 is about 40, which is a little larger than the value for sintered silicon nitride, $n \approx 15$ to 40 [6]. The discrepancy for the values of n may come from the fact that Y and σ are taken as constants and that n for Equation 11 is microscopic in contrast with macroscopic for the crack growth measurement. A detailed paper will be given in near future.

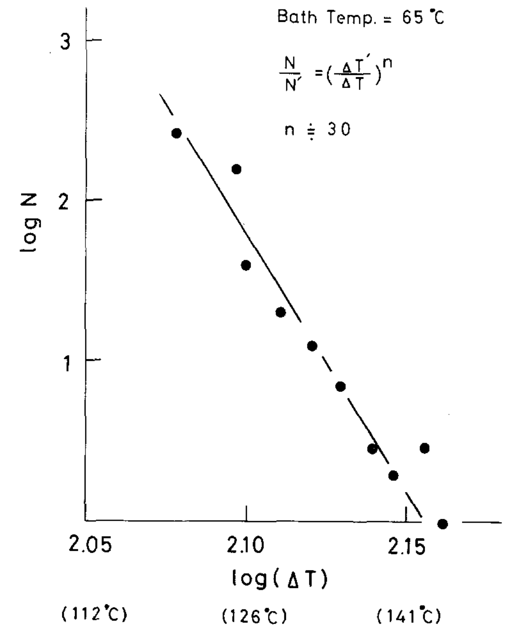

Figure 1 Thermal shock severity (ΔT) versus thermal fatigue life (N) in soda-lime–silica glass rods. (Results from Hasselman *et al.* [4]).

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Received 21 March and accepted 28 April 1977.

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